Quenched Hadron Spectrum and Decay Constants on the Lattice

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Abstract

In this talk we present the results obtained from a study of $\mathcal{O}(2000)$ (quenched) lattice configurations from the APE collaboration, at $6.0 \leq \beta \leq 6.4$, using both the Wilson and the SW-Clover fermion action. We determine the light hadronic spectrum and the meson decay constants. For the light-light systems we find an agreement with the experimental data of $\sim 5\%$ for mesonic masses and $\sim 10\% - 15\%$ for baryonic masses and pseudoscalar decay constants; a larger deviation is present for the vector decay constants. For the heavy-light decay constants we find $f_{D_s} = 237 \pm 16$ MeV, $f_D = 221 \pm 17$ MeV ($f_{D_s}/f_D = 1.07(4)$), $f_{B_s} = 205 \pm 35$ MeV, $f_B = 180 \pm 32$ MeV ($f_{B_s}/f_B = 1.14(8)$), in good agreement with previous estimates.

Introduction

The aim of this exposition is to describe the recent results obtained from the APE collaboration on the hadronic spectrum and the meson decay constants. A full description of the method used and the results obtained are reported in [1, 2].

The lattice technique has proved a very effective theoretical tool to determine phenomenological quantities such as the mass spectrum and weak decay matrix elements. Unlike other approaches, it does not (in principle) suffer from uncontrolled approximations. However, in practice, one is forced to work on a lattice of (i) finite size, with (ii) a finite lattice spacing, and (iii) unphysically large masses for the light quarks. Also the quenched approximation is often used in lattice studies.

The aim of the high statistics simulations [1, 2] is to study the main systematic errors present in the extraction of the light hadronic spectrum and meson decay constants. It is important to study the systematics due to (i), (ii) and (iii) before the effects of the quenched approximation can be correctly understood.

The results reported in this talk are obtained from $\mathcal{O}(2000)$ (quenched) lattice configurations from the APE collaboration, for different lattice volumes at $6.0 \leq \beta \leq 6.4$ using both the Wilson action and the SW-Clover fermion action. The main parameters used in each simulation are listed in table 1. The values of beta have been chosen:

a) small enough to obtain accurate results on reasonably large physical volumes;

b) large enough to be in the scaling region.

In this range of β we cannot draw any conclusion about a dependence of hadron masses and pseudoscalar decay constants. On the other hand, we find that the quenched approximation gives a resonable agreement with the experimental values, when the comparison is possible.

The main physical results of our study have been given in the abstract.

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Table 1: Summary of the parameters of the runs analyzed.

	C60b	C60a	W60	C62a	W62a	C62b	W62b	W64	C64
β	6.0	6.0	6.0	6.2	6.2	6.2	6.2	6.4	6.4
Action	SW	SW	Wil	SW	Wil	SW	Wil	Wil	SW
# Confs	170	200	320	250	250	200	110	400	400
Volume	$18^3 \times 64$	$18^3 \times 64$	$18^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$18^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$
K_l	-	-	-	0.14144	0.1510	-	-	0.1488	0.1400
	0.1425	0.1425	0.1530	0.14184	0.1515	0.14144	0.1510	0.1492	0.1403
	0.1432	0.1432	0.1540	0.14224	0.1520	0.14190	0.1520	0.1496	0.1406
	0.1440	0.1440	0.1550	0.14264	0.1526	0.14244	0.1526	0.1500	0.1409
k_H	0.1150	-	0.1255	0.1210	0.1300	-	0.1300	-	-
	0.1200	-	0.1320	0.1250	0.1350	-	0.1350	-	-
	0.1250	-	0.1385	0.1290	0.1400	-	0.1400	-	-
	0.1330	-	0.1420	0.1330	0.1450	-	0.1450	-	-
	-	-	0.1455	-	-	-	0.1500	-	-
	Light-light mesons with zero momentum								
$t_1 - t_2$	15-28	15-28	15-28	18-28	18-28	18-28	18-28	24-30	24-30
	Heavy-light mesons with zero momentum								
$t_1 - t_2$	15-28	15-28	15-28	20-28	20-28	-	20-28	-	-
	Baryons with zero momentum								
$t_1 - t_2$	-	12-21	12-21	18-28	18-28	18-28	18-28	22-28	22-28
$a_{K^*}^{-1}$	2.00(10)	1.98(8)	2.26(5)	2.7(1)	3.00(9)	3.0(3)	3.0(1)	4.1(2)	4.0(2)

Lattice Details

Hadron masses and decay constants have been extracted from two-point correlation functions in the standard way. For the meson masses and decay constants we have computed the following propagators:

$$\begin{aligned} G_{55}(t) &= \sum_x \langle P_5(x,t) P_5^\dagger(0,0)\rangle \ , \\ G_{05}(t) &= \sum_x \langle A_0(x,t) P_5^\dagger(0,0)\rangle \ , \end{aligned} \tag{1}$$

where

$$\begin{aligned} P_5(x,t) &= i\bar{q}(x,t)\gamma_5 q(x,t) \ , \\ A_\mu(x,t) &= \bar{q}(x,t)\gamma_\mu\gamma_5 q(x,t) \ . \end{aligned}$$

and the following propagators of the vector states:

$$G_{ii}(t) = \sum_{i=1,3}\sum_x \langle V_i(x,t) V_i^\dagger(0,0)\rangle \ , \tag{2}$$

where

$$V_i(x) = \bar{q}(x,t)\gamma_i q(x,t) \ .$$

In order to determine the baryon masses we have evaluated the following propagators:

$$\begin{aligned} G_n(t) &= \sum_x \langle N(x,t) N^\dagger(0,0)\rangle \ , \\ G_\delta(t) &= \sum_x \langle \Delta_\mu(x,t) \Delta_\mu^\dagger(0,0)\rangle \ , \end{aligned} \tag{3}$$

where

$$\begin{aligned} N &= \epsilon_{abc}(u^a C\gamma_5 d^b)u^c \\ \Delta_\mu &= \epsilon_{abc}(u^a C\gamma_\mu u^b)u^c \ . \end{aligned}$$

We have fitted the zero-momentum correlation functions in eqs. 1, 2 and 3 to a single particle propagator with *cosh* or *sinh* in the case of mesonic and axial-pseudoscalar correlation functions and with an *exp* function in the case of the baryonic correlation functions

$$\begin{aligned} G_{55}(t) &= \frac{Z^{55}}{M_{PS}} \exp(-\frac{1}{2}M_{PS}T)\cosh(M_{PS}(\frac{T}{2}-t)) \ , \\ G_{ii}(t) &= \frac{Z^{ii}}{M_V} \exp(-\frac{1}{2}M_V T)\cosh(M_V(\frac{T}{2}-t)) \ , \\ G_{05}(t) &= \frac{Z^{05}}{M_{PS}} \exp(-\frac{1}{2}M_{PS}T)\sinh(M_{PS}(\frac{T}{2}-t)) \ , \\ G_{n,\delta}(t) &= C^{n,\delta}\exp(-M_{n,\delta}t) \ , \end{aligned} \tag{4}$$

in the time intervals reported in table 1. In eqs. 4, T represents the lattice time extension, the subscripts PS and V stand for pseudoscalar and vector meson, n and δ stand for nucleon- and delta-like baryons[2]. To improve stability, the meson (axial-pseudoscalar) correlation functions have been symmetrized (anti-symmetrized) around $t = T/2$. The time fit intervals have been chosen with the following criteria: we fix the lower limit of the interval as the one at which there is a stabilization of the effective mass, and, as the upper limit the furthest possible point before the error overwhelms the signal.
The pseudoscalar and vector decay constants f_{PS} and $1/f_V$ are defined through the equations

$$\langle 0|A_0|PS\rangle = i\frac{f_{PS}}{Z_A}M_{PS} \ , \tag{5}$$

$$\langle 0|V_i|V,r\rangle = \epsilon_i^r \frac{M_V^2}{f_V Z_V} \ , \tag{6}$$

where ϵ_i^r is the vector-meson polarization, M_{PS} and M_V are the pseudoscalar and vector masses and $Z_{V,A}$ are the renormalization constants. We have extracted f_{PS}

[2]i.e. n stands for the nucleon N, $\Lambda\Sigma$ or Ξ baryon, and δ is either a Δ^{++} or a Ω.

Table 2: Predicted hadron masses in GeV for all lattices, using the scale from M_{K^*}.

	M_ρ	M_ϕ	M_N	$M_{\Lambda\Sigma}$	M_Ξ	M_Δ	M_Ω
Exper.	0.770	1.019	0.9389	1.135	1.3181	1.232	1.6724
C60a	0.809(7)	0.977(7)	1.09(5)	1.21(4)	1.32(4)	1.3(1)	1.60(9)
W60	0.808(3)	0.978(3)	1.19(5)	1.29(4)	1.40(4)	1.46(7)	1.71(4)
C62a	0.81(1)	0.98(1)	1.1(1)	1.22(8)	1.34(7)	-	-
W62a	0.803(6)	0.984(6)	1.17(7)	1.28(6)	1.39(5)	-	-
C62b	0.79(1)	1.00(1)	1.1(2)	1.2(2)	1.4(1)	1.6(3)	1.9(2)
W62b	0.797(7)	0.989(7)	1.2(1)	1.3(1)	1.40(9)	1.50(9)	1.72(5)
W64	0.796(4)	0.990(4)	1.21(9)	1.32(8)	1.43(6)	1.4(2)	1.72(9)
C64	0.792(4)	0.994(4)	1.2(1)	1.29(8)	1.41(7)	1.3(2)	1.7(1)

from the ratio

$$\begin{aligned} R_{f_{PS}}(t) &= Z_A \frac{G_{05}(t)}{G_{55}(t)} \\ &\longrightarrow Z_A \frac{Z^{05}}{Z^{55}} \tanh(M_{PS}(\frac{T}{2}-t)) \\ &= Z_A \frac{\langle 0|A_0|P\rangle}{\langle 0|P_5|P\rangle} \tanh(M_{PS}(\frac{T}{2}-t)) \\ &= \frac{f_{PS}M_{PS}}{\sqrt{Z^{55}}} \tanh(M_{PS}(\frac{T}{2}-t))\ , \end{aligned} \tag{7}$$

and the vector-meson decay constant has been obtained directly from the parameters of the fit to $G_{ii}(t)$, eqs. 4:

$$\frac{1}{Z_V f_V} = \frac{\sqrt{Z^{ii}}}{M_V^2}\ . \tag{8}$$

Light-Light hadron masses and decay constants

In order to reduce the error coming from the chiral extrapolation, we have extracted as much physics as possible from the "strange" region. The method we have used to extract the hadron masses and the light-light mesons decay constants from the lattice data is fully explained in [1]. In tables 2 - 3, we give a list of the lattice predictions for the light mass spectrum and light-light decay constants from C60a - C64. The scale is set from the K^* mass. Note that all errors quoted in tables 2 - 3 are statistical only. As can be seen from table 2, for the hadron masses there is good consistency of the physical predictions between the different simulations within the errors.
To compare the lattice decay constants with the experimental ones, we have used a 'boosted' one-loop form of the renormalization constants [3] - [4]:

$$\begin{aligned} \text{Wilson action} \quad Z_A &= 1 - 0.134 g^2_{\overline{MS}} \\ Z_V &= 1 - 0.174 g^2_{\overline{MS}} \end{aligned}$$

Table 3: Predicted meson decay constants for all lattices, using the scale from M_{K^*}.

	f_π (GeV)	$\frac{1}{f_\rho}$	f_K (GeV)	$\frac{1}{f_{K^*}}$	$\frac{1}{f_\phi}$
Exper.	0.1307	0.28	0.1598		0.23
C60a	0.134(9)	0.35(3)	0.149(8)	0.33(2)	0.30(1)
W60	0.155(7)	0.37(2)	0.167(6)	0.35(1)	0.324(8)
C62a	0.124(9)	0.33(3)	0.143(8)	0.30(2)	0.281(7)
W62a	0.135(6)	0.35(2)	0.153(5)	0.33(1)	0.307(6)
C62b	0.14(2)	0.26(4)	0.16(2)	0.25(3)	0.25(2)
W62b	0.135(8)	0.36(2)	0.152(7)	0.34(2)	0.315(9)
W64	0.147(9)	0.29(1)	0.161(8)	0.283(9)	0.272(6)
C64	0.144(9)	0.25(1)	0.158(8)	0.25(1)	0.243(8)

$$\begin{aligned}\text{Clover action} \quad Z_A &= 1 - 0.0177 g^2_{\overline{MS}} \\ Z_V &= 1 - 0.10 g^2_{\overline{MS}}\end{aligned}$$

where $g^2_{\overline{MS}} = 6/\beta_{\overline{MS}}$, with

$$\beta_{\overline{MS}} = < U_{plaq} > \beta + 0.15. \tag{9}$$

The results are shown in table 3. For the ratio f_{PS}/M_V we see again a global consistency of our data both for Wilson and the SW-Clover action. We do not see a strong a dependence for either actions comparing lattices at different β (C60a, C62a and W60, W62a). The experimental points lie quite well in the extrapolated/interpolated lattice data.

The values of the pseudoscalar decay constants in table 3 show larger deviations from the experimental data than the ratio f_{PS}/M_V. It is clear that a cancellation of systematic error occurs in this ratio. Still there is an agreement within 1.5 standard deviations with the experimental data apart from the W60 value which is quite high.

The situation is more delicate for the vector decay constant for which a dependence on the volume and a^{-1} may be present but with our data it would be difficult to disentangle the two effects [1]. The 'strange' vector decay constants seem to be slightly more stable also because no extrapolation is needed.

Strange Quark mass

Lattice QCD is in principle able to predict the mass of a quark from the experimental value of the mass of a hadron containing that quark. The 'bare' lattice quark mass $m(a)$ can be extracted directly from lattice simulations and can be related to the continuum mass $m^{\overline{MS}}(\mu)$ renormalized in the minimal-subtraction dimensional scheme through a well-defined perturbative procedure [5]. Following ref. [5]

$$m^{\overline{MS}}(\mu) = Z_m^{\overline{MS}}(\mu a) m(a)$$

Table 4: Values for the lattice strange lattice quark masses for all lattices and the corresponding $\overline{MS}$ values at NLO, both in MeV.

	$m_s(a)$	$m_s^{\overline{MS}}(\mu = 2\ GeV)$
C60a	89(3)	120(10)
W60	98(2)	130(20)
C62a	83(4)	120(10)
W62a	93(3)	130(10)
C62b	75(6)	110(10)
W62b	92(4)	130(20)
W64	82(3)	120(10)
C64	69(3)	100(10)

where $m(a)$ is the bare lattice quark mass and $Z_m^{\overline{MS}}(\mu a)$ is the mass renormalization constant at scale μ which we choose to be 2 GeV. The bare and renormalized strange quark masses are reported in table 4. The error on $m^{\overline{MS}}(\mu)$ has been estimated as in ref. [5] taking into account the spread due to different definitions of the strong coupling constant. There is a good consistency among the values coming from the different lattices and we do not see any dependence on the lattice spacing a within the errors in the Wilson data and a mild tendency in the clover data to decrease with increasing β. We therefore conclude that any $O(a)$ effects present are beneath the level of statistics, and/or hidden among finite volume effects. We then extract the average value of the strange quark mass without any extrapolation and get:

$$m_s^{\overline{MS}}(\mu = 2GeV) = 122 \pm 20 MeV$$

which is in agreement with the result of ref. [5]. It is also compatible with the value $m_s^{\overline{MS}}(\mu = 2GeV) = 100 \pm 21 \pm 10\ MeV$ of ref. [6], but one should take into account that this value comes from an analysis on various Wilson and Staggered lattices at different values of β and an extrapolation in a.

Heavy-Light decay constants

The major sources of uncertainty in the determination of the heavy-light pseudoscalar decay constants, besides the effects due to the use of the quenched approximation, come from the calculation of the constant Z_A in eq. (5) and from discretization errors of $O(a)$ present in the operator matrix elements. A method to get rid of Z_A consists of extracting the decay constants of heavier pseudoscalar mesons by computing the ratio $R_P = f_P/f_\pi$ and multiplying R_P by the experimental value of the pion decay constant. Hopefully, by taking the ratio, some of the $O(a)$ effects are eliminated. These effects are expected to be more important for f_D than for $f_{\pi,K}$ since, at current values of a, the relevant parameter $m_{\rm charm}a$ is not very small. A full analysis of the discretization errors is performed in the section 2 of [2]. The results are reported in table 5[3]. We will

[3]For the run W60, the heavy-light correlators are computed on a subset of 120 configurations.

Table 5: Predicted heavy-light meson decay constants for all lattices.

Run	f_{D_s}/f_K	f_D/f_π	f_{D_s}/f_D	f_{B_s}/f_K	f_B/f_π	f_{B_s}/f_B
C60b	1.56(3)	1.63(4)	1.08(1)	1.48(7)	1.53(9)	1.10(3)
C62a	1.48(6)	1.58(8)	1.07(2)	1.28(9)	1.32(13)	1.14(6)
W60[3]	1.11(3)	1.14(4)	1.06(1)	0.79(4)	0.81(5)	1.05(2)
W62a	1.23(4)	1.31(6)	1.07(1)	0.83(4)	0.86(6)	1.10(3)
W62b	1.19(5)	1.25(7)	1.09(2)	0.84(5)	0.86(6)	1.12(3)

first discuss the results for charmed mesons, for which the extrapolation in the heavy quark mass is not a relevant source of systematic uncertainties, and then discuss the B-meson case. We find that higher statistics and larger intervals of a values are required to satisfactorily uncover the a dependence of the decay constants. An extrapolation to $a = 0$ it is not possible at this stage. Thus we believe that the best estimate of f_{D_s} is obtained from the Clover data at $\beta = 6.2$, by using $R_{D_s} = f_{D_s}/f_K$ (from a linear fit in the light quark masses, a quadratic fit in $1/M_{P_s}$ and without any KLM factor). As for the error, we take as a conservative estimate of the discretization error the difference between the results obtained at $\beta = 6.0$ and 6.2, and combine it in quadrature with the statistical one. This gives $R_{D_s} = 1.48(10)$ from which, by using $f_K^{\rm exp} = 159.8$ MeV, we obtain $f_{D_s} = 237 \pm 16$ MeV. This value is in very good agreement with the experimental value [7][4] of branching fraction $B(D_s^+ \to \mu^+\nu_\mu) = (4.6 \pm 0.8 \pm 1.2) \times 10^{-3}$ which corresponds to a decay constant value of $f_{D_s} = (241 \pm 21 \pm 30)$ MeV (note that f_{D_s} was predicted by lattice calculations long before its measurement). By using $R_{D_s} = 1.48(10)$ combined with $f_{D_s}/f_D = 1.07(4)$ we obtain $f_D = 221 \pm 17$ MeV. We believe that these are our "best" results.

In order to obtain f_{B_s} and f_B, an extrapolation in the heavy quark mass well outside the range available in our simulations is necessary. Discretization errors can affect the final results in two ways. Not only they can change the actual values of the decay constants, but also deform the dependence of f_P on m_H. Moreover, points obtained at the largest values of $m_H a$ become the most important, since we extrapolate in the direction of larger values of m_H. A full analysis of these effects is done in [2].

In order to extract our "best" values for f_{B_s} and f_B we have proceed exactly as for the D-meson case. We have obtained $f_{B_s} = 205 \pm 15 \pm 31$ MeV$= 205 \pm 35$ MeV, where the second error (31 MeV) is the discretization error, estimated by comparing the results from C60b and C62a as done for f_{D_s}. We also have obtained $f_B = 180 \pm 32$ MeV. The decay constants of the B-mesons are not yet measured and the numbers given above are predictions of the lattice. The results we have obtained for the heavy-light decay constants are in very good agreement with the compilations of lattice calculations presented in refs. [9, 10].

[4] There is a new measurement of $B(D_s^- \to \tau^-\bar\nu_\tau)$ from L3 [8]. They have obtained $B(D_s^- \to \tau^-\bar\nu_\tau) = (0.074 \pm 0.028({\rm stat}) \pm 0.016({\rm syst}) \pm 0.018({\rm norm}))$ which corresponds to $f_{D_s} = (309 \pm 58({\rm stat}) \pm 33({\rm syst}) \pm 38({\rm norm}))$ MeV.

Conclusions

In this talk we have reported results from a large set of data on lattices of different lattice spacing and lattice sizes obtained with both the Wilson and SW-Clover actions. The results obtained and reported in the abstract give us confidence in the ability of the lattice to predict other non-perturbative quantities that are of phenomenological interest. Further studies, with comparable (or smaller) statistical errors and physical volume, at smaller values of the lattice spacing, are required to reduce the systematic error due to O(a) effects. The systematic errors will be completely understood only when the quenched approximation will be removed.

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